A simplified model for calculating the insertion gain of track support systems using the finite difference method

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Abstract. In the procurement of track support systems for railways planned in the vicinity of receptors sensitive to groundborne noise and vibration it can be necessary to specify parameters of components of the system in order to ensure that its in-service performance will deliver the required level of mitigation against the effects of groundborne noise and vibration. In previous comparable schemes in the UK it has been the practice to specify the performance of the track system in terms of Insertion Gain (IG). This is defined as the difference between the vibration spectrum at a reception point with the mitigating track in place, and with a hypothetically "very stiff" track support system. It is necessary to have a means of verifying that a proposed system is likely to satisfy the requirements and it is desirable to simplify the method of evaluation of predicted track performance based on the stated parameters. A simplified algorithm for rapidly calculating the insertion gain of a two-stage (or single-stage) resilient track support system has been developed using the finite-difference method. This provides a tool for evaluating the predicted performance of a variety of proposed track support systems in order to compare it against the required specification.

Keywords: Insertion gain, finite difference, vibration isolation.

1 Introduction

The High Speed 2 project in the UK will run in tunnels beneath residential receptors in a number of locations, in a variety of lithologies. Prediction of the likely level of groundborne noise and vibration has been made using an empirical model at the Environmental Assessment stage [1,2], and a three-dimensional finite-difference-time-domain (FDTD) model in subsequent design stages [4,4].

The FDTD model includes detailed representation of the vehicle, track support system, tunnel and surrounding lithology, and may also include buildings. Track support systems were considered for the purpose of determining the characteristics of the track system that would deliver the project's ground borne noise and vibration criteria. This paper focuses on how these characteristics are specified to a track system supplier.

A rudimentary acoustic performance specification for track would be to place responsibility for achieving numerical commitments relating to groundborne noise and vibration on the track supplier. The prediction of ground-borne noise and vibration in buildings above the tunnels is however complex. Noise and vibration are affected by factors which are outside of the control of the track supplier, such as the characteristics of the rolling stock, tunnel, ground and receiving building.

An alternative is to separate the acoustic performance of the track from the source-path-receiver system. This was achieved on the High Speed 1 [5] and Crossrail [6] Projects in the UK by specifying the acoustic performance of the track as an 'insertion gain', the difference, in decibels, between the amplitude of vibration at a constant receptor when one track support system is replaced by another. The insertion gain enables the acoustic performance of the track to be characterized without the need to specify the performance of the individual components of the track which contribute to the overall performance. Specification as an insertion gain will lead a track system supplier to design and test the key parameters that determine the acoustic performance of the track and then calculate the insertion gain of the track system from these key parameters. Comparison with the insertion gain specification will demonstrate that the track will either achieve, worsen or improve on the ground borne noise performance that was modelled in the full detailed model of the source-path-receiver system.

While the insertion gain of a track support system can be calculated using the full FDTD model by comparing the output for the case in question with a hypothetical case involving a "very stiff" track, it is desirable to simplify the method to remove parameters which cannot be influenced by the track supplier (such as ground conditions or the tunnel design) and to create simplified code for rapid evaluation of predicted track performance.

Previous models have employed 2.5D or 3D numerical models [7] or have [8,9] approximated the track with beam, spring and mass elements coupled to an elastic half-space, or [10] with the track system represented by its impedance per unit length of track. Software for the calculation of vibration from surface and underground railways has been developed [11] which is capable of computing vibration level difference for different track systems, assuming rail pads as continuous spring elements and sleepers as a continuous mass. The method described here models track support as discrete pads, baseplates, blocks or sleepers as appropriate using a simplified finite-difference algorithm and calculates the insertion gain from the difference in the dynamic forces below the track supports.

2 Definition of Insertion Gain

In the railway context, the insertion gain of a track support system is the difference, in decibels, between the amplitude of vibration at a constant receptor when one track support system is replaced by another, expressed as a spectrum (e.g. in 1/3 octaves). While this may refer to an actual track replacement, it may also be used to express the predicted vibration that occurs with one track system in comparison with a hypothetical base case.

3 Principles for calculating an insertion gain

3.1 General

A rail vehicle travelling on a track is a multi-degree of freedom dynamic system, the input signal to which occurs at the contact patch between the wheel and the rail. Looking upwards from the contact patch there is, in simple terms, a spring-mass-spring-mass-spring-mass system (if low frequency bending modes for the vehicle body are ignored). These springs and masses are: the Hertzian contact spring, the wheel and bogie unsprung mass, the primary suspension, the bogie sprung mass, the secondary suspension and the vehicle body. Looking downwards from the contact patch there is a complex mass-spring-mass-spring-mass/spring/resistance in which the components are the rail, the rail foot pad, the sleeper or block, the sleeper support (ballast or a resilient boot/pad) and the foundation in the form of tunnel invert/lining/soil/rock. In a mass-spring-mass-spring system, the result is two coupled natural frequencies, one of which is higher that the higher of the individual mass-spring frequencies and the other is lower than the lower of the individual mass-spring frequencies.

An important example of what happens when a sequence of masses and springs are coupled together is the behaviour of the Hertzian contact spring, which is further discussed in the next section.

3.2 Key parameters

The key parameters that affect the insertion gain of a track system are the geometry of the track system (such as fastener spacing), the mass of the rails, mass of the sleepers (if present) and the acoustic stiffness of the rail or sleeper supports.

Other parameters will affect the insertion gain, such as the mass and suspension of the rolling stock, the Hertzian contact stiffness at the rail-wheel interface, the tunnel and the ground below the track. These are parameters that the track system supplier will have no control over. When comparing track systems these parameters should be kept constant or their effects removed from the insertion gain.

4 Methodology

The methodology started with a process of simplification of the full 3-dimensional model in order to observe the effect associated with a series of simplifications. These included (1) representing the track as a single rail, (2), replacing the line of axles with a single axle, (3) omitting the bogies and vehicle bodies, (4) removing the influence of the lithology (5) removal of the influence of the tunnel lining and (6) assuming frequency-independent damping. The effects associated with these simplifications was identified and is illustrated in Figure 1 which shows the insertion gain calculated using the full source-path-receiver model compared to the simplified models.

A difficulty encountered with the use of a fully detailed model of the system arose when modelling the “very stiff” track support system as a reference case. The effective rail mass in a resiliently supported system is much less than the wheel mass, and the natural frequency associated with the rail-contact-wheel system is close to the mass-spring natural frequency of the rail and the Hertzian spring, typically over 700Hz, and is above the frequency range of interest. In the case of very stiff rail support the natural frequency becomes close to that of the wheel and the Hertzian spring and is well within the frequency range of interest, typically about 180Hz. Consequently the spectrum of vibration transmitted into the foundation by a train on a very stiff track will have a prominent peak within the frequency range of interest and reduced levels of vibration above that frequency. This peak gives a false indication of the performance of the track support, since it is not attributable to the behaviour of the track support system in a relevant way. The means of overcoming this problem, which occurs due to the choice of a reference system that is effectively rigid and does not represent any actual real-world track support system, was to raise the stiffness of the contact patch in the rigid system to a hypothetical value many times greater than its true value, so that the contact-wheel natural frequency is raised to a value above the frequency range of interest.

Having simplified the model in this way, it was also possibly to simplify the finite-difference code to fewer than 500 lines which could be ported to any convenient language. C++ was used in this case. The approach taken was to apply a displacement to the Hertzian contact spring at the wheel/rail contact patch, and to integrate the resulting force at the base of all the lowest resilient elements along the track, which was extended in length so as to be beyond the 10dB down-points for the track decay rate. The relaxation damping algorithm used in the full 3D model was retained in order to be able to model frequency-dependent (or frequency-independent) damping.

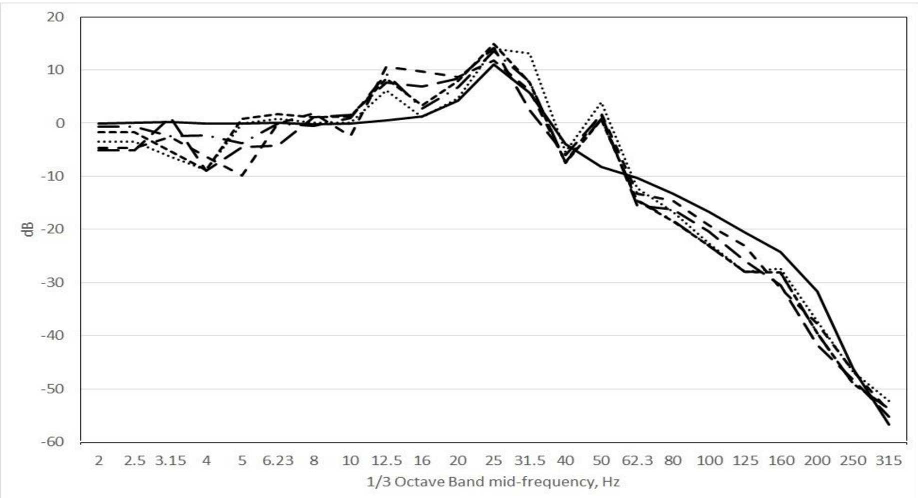

Fig. 1. The effect of simplification: — — Full model; - - one rail; — · — one rail, one axle; - - one rail, one axle, unsprung mass only; ···· one rail, one wheel, unsprung mass only, frequency-independent damping; —— one rail, one wheel, unsprung mass only, 1D model.

4.1 The simplified finite-difference code

The finite difference method was employed in order to solve the bending wave equation for a Euler beam

$$EI\, \partial^4 \zeta / \partial x^4 = -m'\, \partial^2 \zeta / \partial t^2 \quad (1)$$

Where E is the Young's modulus of the beam and I is the second moment of area of the section; m' is the mass per unit length; ζ is the vertical displacement; x is distance along the beam and t is time. This equation neglects rotary inertia and shear, which have an effect on the phase velocity of bending waves in a beam of less than 10% if the wavelength is greater than $6h$ where h is the height of the beam.

The finite-difference approximation of equation (1) is obtained by dividing the rail into 2000 elements approximately 200mm in length (the exact value depending on the fastener spacing to be studied) and for each element of the rail iterating the following two equations

```
ddx4=(((a[i+2]-a[i+1])-(a[i+1]-a[i]))-((a[i+1]-a[i])-(a[i]-a[i-1])))-
(((a[i+1]-a[i])-(a[i]-a[i-1]))-((a[i]-a[i-1])-(a[i-1]-a[i-2])))        (2)
v[i]=v[i]-ddx4*EIdx4                                                   (3)
```

where `i` is the number of the rail element; `a[i]` is the vertical displacement of the rail and `v[i]` is the vertical velocity.

In the model described here, it is assumed that the rail is supported under every third element by a pad of stiffness `kpad`, below which is a block of mass `mblock`. Each block in turn is supported by a resilient boot of stiffness `kboot`.

Following the execution of lines (2) and (3) for each element `i`, the force exerted by the deflected rail pad at every third element is computed in terms of the incremental velocity as

```
dvr=-(a[i]-ab[i])*kpad/mrail/dx*dt
dvb=((a[i]-ab[i])*kpad-ab[i]*kblock)/mblock*dt                         (4)
```

where `ab[i]` is the vertical displacement of the block, `dvr` is the incremental velocity of the rail and `dvb` is the incremental velocity of the block and `dt` is the finite difference time step.

Above the rail is the wheel, treated as a lumped mass on the Hertzian contact spring, with the lines

```
vw=vw-(aw-a[i])*kpatch/mwheel*dt
v[i]=v[i]+(aw-a[i])*kpatch/mrail/dx*dt
aw=aw+vw*dt                                                            (5)
```

where `kpatch` is the stiffness of the Hertzian contact patch.

Damping is modelled using Boltzmann's relaxation model [12]

$$\sigma(t) = D_1\varepsilon(t) - \int_0^\infty \varepsilon(t - \Delta t)\varphi(\Delta t)d(\Delta t) \quad (6)$$

Where $\sigma(t)$ is the stress at time t, $\varepsilon(t)$ is strain, $\varphi(\Delta t) = \frac{D_2}{\tau} e^{-\Delta t/\tau}$, D_1 and D_2 are constants and τ is a relaxation time.

Equation 6 may be replaced by a sum of three terms with different relaxation times, and by choice of values of D_2 and τ any desired frequency-dependence of the resulting damping effect can be achieved.

To implement equation 6 the lines in the code for the rail are

```
de=-(a[i]-ab[i])*kdamp/mrail/dx*dt                          (7)
sp[i][0]=sp[i][0]*phi0+de*dt;
sp[i][1]=sp[i][1]*phi1+de*dt;
sp[i][2]=sp[i][2]*phi2+de*dt;
dv=dv-sp[i][0]*D2/tau0-sp[i][1]*D2/tau1-sp[i][2]*D2/tau2
v[i]=v[i]+dv                                                (8)
```

with corresponding lines for the block.

The use of a length of rail of the order of 400m means that the ends of the rail may be connected end-to-end to avoid reflections and maintain stability, and the length of the modelled rail is long enough for the bending waves to have decayed to insignificant amplitudes at the ends. The time step used was 1/131072 seconds, and the routine was run for 262144 iterations, i.e. 2 seconds. The run time is only a few seconds.

Excitation of the system is achieved by a boundary condition at t=0 consisting of a finite displacement of the Hertzian contact spring, i.e. a Heaviside step.

Output takes the form of the force under the boot below each block, as a time series. Each of these series is then subjected to FFT and the squares of the forces at each block are summed to give a total output force as seen by the rigid support below the boots along the full length of the rail. This is subtracted from the force curve for the rigid system to give the insertion gain (IG). The force curve for the rigid system is found from the IG at frequencies well below the loaded track natural frequency, and has a straight line slope of -9dB per octave.

5 Results

The results take the form of 1/3 octave band spectra of IG, which show the eigenmodes of a coupled two-stage system, taking account of the effect of damping, and the slopes to the insertion gain curves between and above the frequencies of the eigenmodes. For a single stage system, such as slab track with periodic rail supports, the code omits the lines for the blocks and boots. Examples are given in Figure 2. In the results for two-stage systems, the characteristic loaded track resonance is clearly evident, and the coupled natural frequency of the blocks between the rail pads and the resilient boots also appears, reducing the insertion gain around 160Hz-250Hz in the examples shown in Figure 2. This effect is absent in the case of the single stage slab track, but the enhanced attenuation that occurs above the coupled frequency in the two-stage systems is also absent.

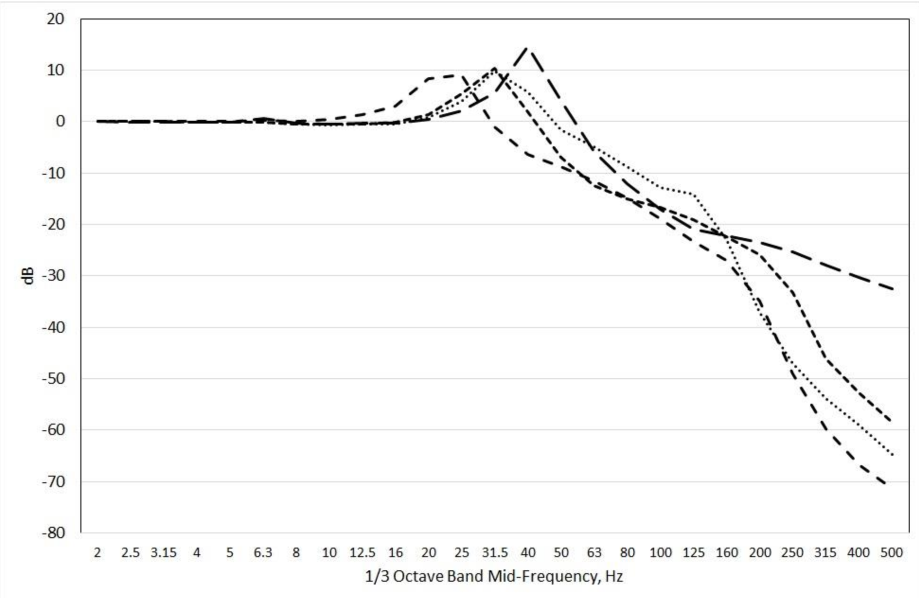

Fig. 2. Simplified IG of track support systems, dB re rigid track: — — slab track; - - booted block low mass; –– –– booted block low stiffness, high mass; ···· booted sleeper.

The properties assigned to each of the cases in Figure 2 were as follows.

Table 1. Input parameters for results in Figure 2.

Track type	Rail pad stiff-ness kN/m	Rail pad damping (dimen-sionless)	Block mass kg	Boot stiff-ness kN/m	Boot damping (dimen-sionless)	Rail mass kg/m	Fastener spacing, m	Wheel mass kg
Slab track	1000	0.15	-	-	-	60	0.25	950
Booted sleeper	150	0.2	210	31.5	0.2	60	0.75	450
Booted block low mass	100	0.2	45	28	0.2	60	0.6	880
Booted block, high mass	200	0.2	120	14	0.2	60	0.6	880

6 Conclusion

The work has led to the development of an open-source computer code for rapidly calculating the insertion gain of a two-stage (or single-stage) resilient track support system. This provides a tool for evaluating the predicted performance of a proposed track support systems in order to compare it against the required specification.

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